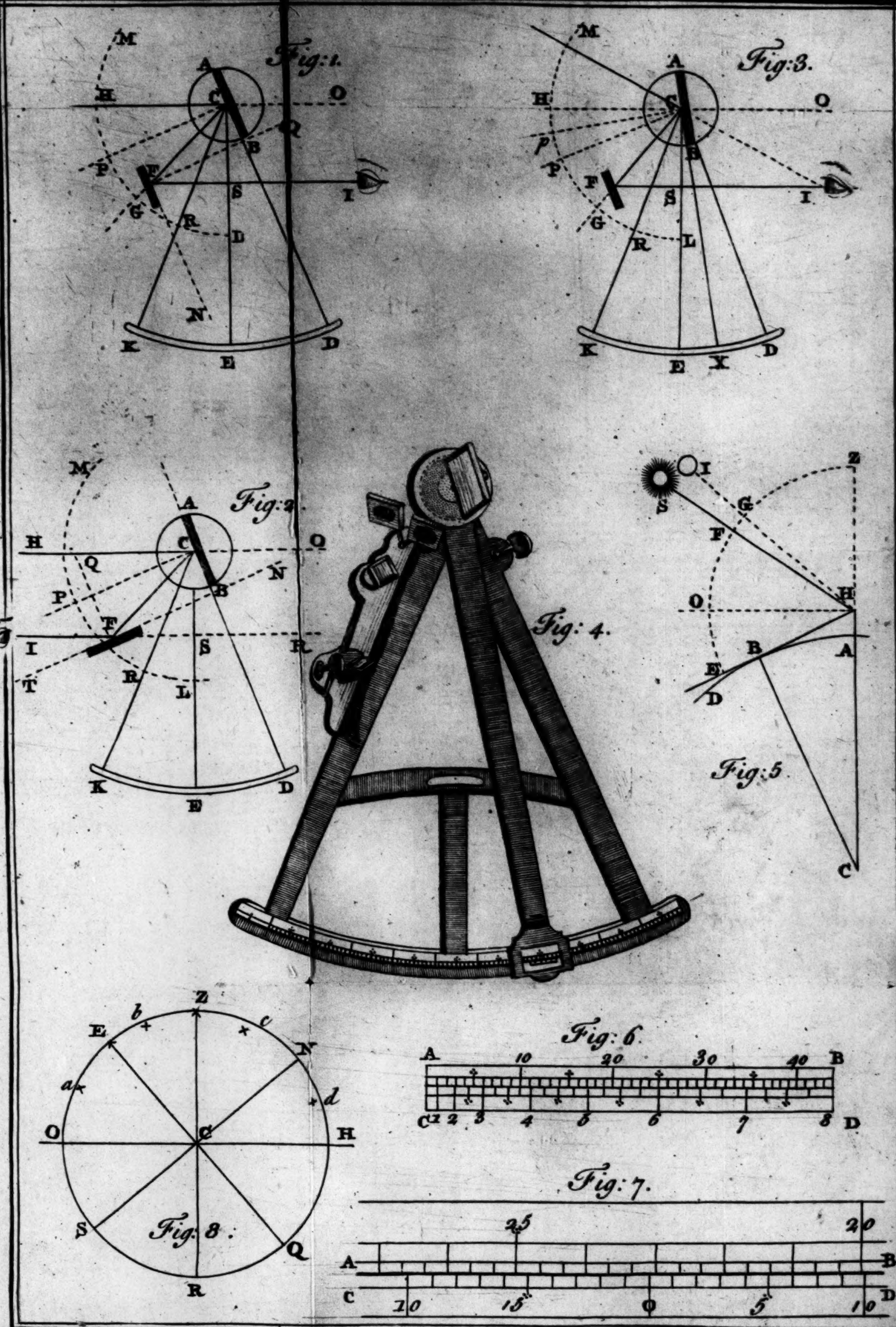
The THEORY *of* HADLEY'S QUADRANT *demonstrated.*



THE

THEORY

OF

HADLEY'S QUADRANT Demonstrated;

And from thence

Its NATURE, CONSTRUCTION, and USES are fully shewn.

With a Table of

The SUN'S DECLINATION

For finding

The LATITUDE of the PLACE.

To which is added,

A New Construction of the Quadrant, which, by Means of an *artificial* HORIZON, renders it of universal Use by SEA and LAND.

By BENJ. MARTIN.

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PREFACE.

I Have always looked upon the PRACTICE *of any* Art *as very deficient without the* THEORY; *and the ſame may be ſaid with regard to the Uſe of any Inſtrument; for as Errors in Practice are oftentimes unavoidable by the moſt ingenious Artiſt, and cannot be corrected without the Theory; ſo if any Fault happen in the Conſtruction of an Inſtrument, it cannot be detected ſo well as by a Knowledge of the* Rationale *thereof, or of the Reaſon of every Part which compoſes it, and points out its Uſe. Nor is there any Art of greater Importance than* NAVIGATION, *nor any Inſtrument uſed in this Art of ſo much Conſequence as* Hadley's Quadrant, *(originally invented by Sir* I. Newton.) *The* Theory *of this Inſtrument, therefore, ought, above all Things, to be underſtood by every* Mariner *who has any Share in conducting a Ship. But as I have not ſeen any Book where it is explained to the Capacity of young* Navigators, *I thought it would not be miſapplying Time, if I offered him my Aſſiſtance in drawing up a plain and eaſy Demonſtration of the Nature, Conſtruction, and Uſe of this capital Inſtrument, and which I here preſent them with in the following*

ſmall Tract. I have alſo added a Table of the Sun's Declination, for finding the Latitude of the Place by the obſerved Meridian Altitude, and which ought to go with every Quadrant that is ſold, to render its Uſe compleat.

As Sea-faring Men *make that Claſs of People on whom a* Maritime State *has the greateſt Dependance, (and no Nation is ſo conſpicuous in that Character as* GREAT-BRITAIN) *they ought, among us, to be diſtinguiſhed with proper Regard; to be on all Occaſions humanely treated; to have due Encouragement and adequate Rewards; and to have all the Helps afforded them to render their arduous Practice, in navigating a Ship, as eaſy and pleaſant as poſſible. I ſhould be glad if it were more in my Power to be ſubſervient in promoting that great End. This is not the firſt, nor (probably) will be the laſt Inſtance of my Willingneſs and Readineſs to contribute thereto;*

Who am,

With real Sincerity and Pleaſure,

Their moſt obedient Servant,

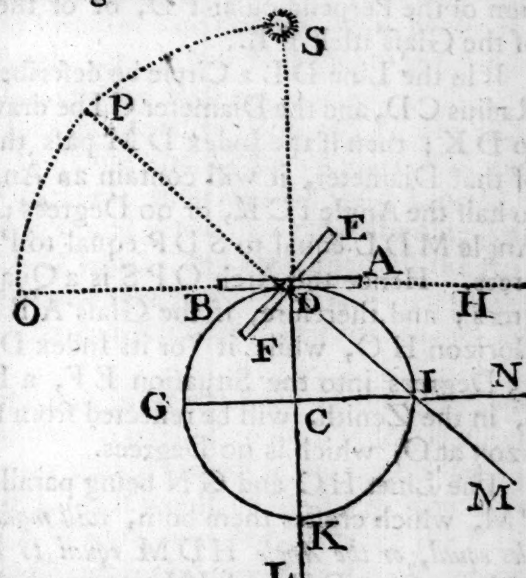
BENJ. MARTIN.

The

The THEORY *of* HADLEY's QUADRANT *demonstrated.*

IT is not to be ſuppoſed that any Perſon can underſtand the Uſe of any optical Inſtrument whatſoever, who is not acquainted with the *firſt Principle* (or rather *Axiom*) of OPTICS, *viz.* That in Caſe of Light falling on, and reflected from any poliſhed Surface, or *Speculum, the Angle of Incidence is ever equal to the Angle of Reflection.*

This Principle is demonſtrated by *Writers* on OPTICS, both from Reaſon, and Experiment; and is eaſily illuſtrated by the annexed Figure.



Let E F be a Speculum, or plain Looking-glaſs, on which a Ray of Light S D falls in the Point D; then if the Line P D be perpendicular to the Glaſs in the Point D, and make the Angle P D O equal to the Angle P D S, the Line D O will be the *reflected Ray*; and in all Caſes, the *Angle of Incidence* S D P *will ever be equal to the Angle of Reflection* P D O.

If D M be an Index fixed at Right-angles to the back Part of the Speculum F E in the Point D, it will then be in the Direction of the Perpendicular P D; and ſuppoſing the incident Ray S D continued from D to L, then by

inclining or moving the Index D M towards D L, the Glaſs F E will have the ſame Motion on the Point or Center D, and at Length arrive to the Situation A B, during which Time the Perpendicular P D will gradually approach to the incident Ray S D, and, at laſt, coincide with it.

But as the Perpendicular P D approaches the Ray S D, the Angle of Incidence P D S is conſtantly decreaſing, and therefore, alſo, the Angle of Reflection P D O, which is always equal to it. And when P D coincides with S D, or D M with D L, then both thoſe Angles vaniſh; and the incident Ray is reflected back upon itſelf, or in the ſame perpendicular Direction from D to S.

In the Time the Perpendicular P D moves through the Arch P S, the reflected Ray D O moves thro' the Arch O S, which is double the Arch P S. Therefore the Motion of the reflected Ray O D is twice as quick as the Motion of the Perpendicular P D, or of the Index D M, or of the Glaſs itſelf F E.

If in the Line D L a Circle be deſcribed on C, with the Radius C D, and the Diameter G I be drawn at right Angles to D K; then if the Index D M paſs through the End I of that Diameter, it will contain an Angle M D L equal to half the Angle I C K, or 90 Degrees; therefore the ſaid Angle M D L equal to S D P equal to P D O, is 45 Degrees. Hence the Arch O P S is a Quadrant, or 90 Degrees; and therefore, if the Glaſs A B be parallel to the Horizon H O, while it (or its Index D M) moves thro' 45 Degrees into the Situation E F, a Ray from the Sun S, in the Zenith, will be reflected from thence to the Horizon at O, which is 90 Degrees.

The Lines H O and G N being parallel, the ſtrait Line P M, which croſſes them both, *will make the alternate Angles equal, or the Angle* H D M *equal to* D I G; for it is evident, ſince D H and I N are parallel, they make the Angle H D I equal to the Angle N I M; but the Angle N I M is equal to the Angle D I G; therefore the Angle H D I is equal to the Angle D I G.

Every Thing we have hitherto premiſed, is, we preſume, evident from a bare Inſpection of the Figure, and is ſufficient for underſtanding what we have further to ſay rela-

relating to the Reflection of Light from two Glasses, as they are put together in the Construction of *Hadley's Quadrant*, and from whence the *Rationale* of that Instrument will fully appear.

Let A B (Fig. 1.) represent the larger Glass of *Hadley's Quadrant*, moveable about the Center C of the Instrument, by Means of the Index on which it is fixed; and let P C be perpendicular thereto in the Point C; then if any Ray of Light, as H C, fall upon it in the Point C, it will be so reflected as to make the Angle of Incidence H C P equal to the Angle of Reflection P C G; and therefore, by describing on the Center C the Circle M G L, and making the Arch P G equal to P H, the Line C G will be the reflected Ray.

Again; let F be the smaller Glass in a Position F N, parallel to that of the larger one A B, and suppose F Q perpendicular thereto; then, again, the Ray of Light C G falling on this Glass on the Point F, will be so reflected, that the Angle C F Q will be equal to the Angle Q F I, if F I be the reflected Ray.

Now because the two Glasses A B, and F, are in a parallel Position, therefore the Perpendiculars P C and F Q will also be parallel; therefore the right Line C F which crosses them in C and F will make the alternate Angles P C F and C F Q equal; and consequently, the Angle H C P will be equal to the Angle Q F I, which proves *that the Ray* F I, *after two Reflections, is parallel to the incident Ray* H C.

Hence it follows, that in all Positions of the Quadrant (while the Glasses remain parallel to each other) the Eye at I will view any very distant Object in the same Place by the reflected Ray F I, as it would do by the incident Ray at O, if the Glass A B were removed. Hence the Reason of adjusting the Quadrant by a *distant Object*, either the Horizon itself, or any other remote Object, since in such a Case, the Object seen by the direct Ray H C, and the Image shewn by the reflected Ray F I do coincide, or appear in one and the same Place.

But this will not be the Case when the Object is so near, that the Line or Distance C S subtends at the Object a sensible Angle, because then the Incident and reflected

Rays H C and F I interſect each other; and therefore the Object, and Image in the Glaſs at F, will appear in two different Places. The leaſt Diſtance of an Object, by which the Glaſſes can be truely adjuſted, is about half a Mile; but farther off the better.

If the Poſition of the ſmall Glaſs (Fig. 2.) F be perpendicular to that of the large Glaſs A B, or T F N perpendicular to A B D, and the Line Q F perpendicular to the ſaid Glaſs in F; then the Angle H C P is equal to the Angle P C F, as before; and alſo the Angle C F Q is equal to the Angle Q F I; therefore the Angle C F N is equal to the Angle I F T; but T N is parallel to P C; therefore the Angle C F N is equal to the Angle F C P, and alſo to the Angle P C H; conſequently the Angle T F I is equal to the Angle P C H. But T F is parallel to P C, therefore the reflected Ray I F is parallel to the incident Ray H C. Wherefore the Image of an Object, at a great Diſtance, is ſeen at R, by an Eye at I in the ſame right Line with the Object itſelf, but in an oppoſite Part of the Hemiſphere; which gives the Reaſon *why the Second, or lowermoſt of the ſmall Glaſſes is placed in that Poſition on the Side of the Quadrant for the Back-obſervation.*

If while the Glaſs F retains its Poſition (as in Fig. 1.) the large Glaſs A B be moved on the Index from its parallel Poſition in A D to any other as A X (Fig. 3.) then while the Index C X deſcribes the Angle D C X, the Perpendicular P C will deſcribe the equal Angle P C *p*. In this Situation of the Glaſſes, let M C be the incident Ray, and F I the laſt reflected One, produce M C till it interſects the reflected Ray in I; then will the Angle M I F be equal to twice the Angle D C X.

For in this Caſe, the Angle by the firſt Reflection is increaſed from G C H to G C M, by the Exceſs H C M; but the Half the Angle G C M is G C *p*, and its Exceſs above G C P (the Half of G C H) is the Angle P C *p*; therefore the whole Exceſs H C M of the whole Angle G C M above G C H muſt be equal to twice the Angle P C *p*, or twice the Angle D C X; but ſince H O is parallel to F I, the Angle M C H is equal to the Angle M I F; *conſequently, the Angle* M I F *is equal to double the Angle* D C X; *therefore the Angle* M C H (equal to M I F) *is alſo double the Angle* D C X. Or

Or thus; since the Glass F is immoveable, and also the Point C, which is the Center of the moveable Glass A B; it is easy to understand that the Rays C F and F I will each of them be fixed; and therefore the Ray C F may be considered as falling on the Glass A B in the Point C, and thereby reflected in the Direction C M; but C M, considered as the reflected Ray, has twice the Motion of the reflecting Glass A B, or its Index A X; that is, while the Index moves from D to X, the Ray is reflected from H to M, and the Arch H M is double the Arch D X, as we shewed in the Beginning was always the Case in a single Reflection.

Hence it appears, that by moving the Index over any Number of Degrees on the Limb of the Quadrant, you measure an Arch in the Heavens just equal to twice that Number of Degrees. And therefore the *Octant*, or *eighth Part* of a Circle, is, by this Construction, rendered equivalent to a *Quadrant* or *fourth Part* of a Circle for measuring Angles.

But the greatest Advantage of this Instrument, and by which it excels all others for Sea-use, consists in this, *that the Image of an Object, by the second Reflection, is quiescent, or at Rest, while the Quadrant is in Motion*; I mean, that Motion which is made in a vertical Plane passing through the Object. For let the Instrument have what Position it will in that Plane, the Angle M I F is not thereby affected or altered, and consequently the Position of the last reflected Ray is the same, and therefore the apparent Place, or Image of the Object, must necessarily be invariable or at rest.

To demonstrate this in the easiest Case, we need only consider, that as the Quadrant moves on the Point C, or Axis of the Glass A B (Fig. 1.) the Perpendicular P C will be carried to, or from the fixed Line H O, and thereby the Angle P C H becomes diminished or encreased; but the Angle P C F will ever be equal to P C H; and because the Perpendicular Q F is always (in this Case) parallel to P C, therefore the Angle C F Q is equal to P C F; and consequently C F I is always equal to F C H; therefore the reflected Ray F I is parallel to H C constantly, and will of Course shew the Object ever in the *same*

Place,

Place, or *at Rest*. The Reasoning is the same for Fig. 2, and 3. or any other Position of the Glasses, provided they are parallel to each other when the Index is at the Beginning of the Degrees on the Limb of the Quadrant.

If the Quadrant librates Sideways, or has a Motion contrary to the former, there will, indeed, be a Motion of the Image produced, because, since the Image is always formed in the Plane of Reflection, which passes thro' the Object and the Eye of the Spectator; it is evident, if that Plane be changed, the Place of the Image must change with it, and a Motion of the Image will be the Effect of such a Motion of the Instrument. But then this Motion is so far from being detrimental, that on the contrary it is of very great Use in many Cases, which the Mariner very well knows.

These are all the essential Particulars which constitute the Nature and Theory of this capital Instrument; the Form of which, as it is fitted for Use, with a *Nonius*, is that in Fig. 4.

Directions for the Use of HADLEY'S QUADRANT *at* SEA, *with the Rationale of each particular Process.*

THE Quadrant being applied in the usual Manner to take Altitudes by reducing all celestial Objects to the Level or *Edge of the Sea*, this we may call the *Marine Horizon*, in Contradistinction to the *true* or *natural Horizon*.

To explain this Matter (Fig. 5.) let A B D be a Part of the Surface of the Sea; C, the Center of the Earth; A C, its Semidiameter equal to 43946552 *English* Feet. Then if A H be any Height to which the Eye is elevated above the Surface of the Sea at A, and through the Point H you draw H O perpendicular to H C, that Line H O will be the *true Horizon:* And if from the Point H you draw H E to touch the Surface of the Sea in some Point

Point B; then is the Line HE the *Marine Horizon*, or that which appears to the Eye. Laſtly, the Angle contained between the two, *viz.* the Angle O H E is called the *Dip of the Marine, or viſible Horizon.*

Hence when the Altitude of the Sun or Star at S is taken by the Quadrant, the Angle meaſured on the Limb is S H E, greater than the true Altitude S H O, by the ſaid Dip, or Angle O H E, which therefore muſt be ſubducted from the meaſured Altitude in the *fore Obſervation.*

But in the back Obſervation, the Dip of the Sea, or Angle O H E is to be added to the meaſured Altitude, becauſe the *Phænomenon* does in this Caſe apparently aſcend, and comes to the Marine Horizon before it comes to the true One.

The Quantity of the Dip, or Angle O H E is computed from the Triangle B C H, right angled at B, in which the Side B C is known, and alſo the Side C H when the Height of the Eye A H is given.

Thus when A H is	44	Feet, the Angle	O H E is	8 Minutes,
	34	——	——	7
	25	——	——	6
	17	——	——	5
	11	——	——	4
	6	——	——	3
	3	——	——	2
	1	——	——	1

which correſponding Meaſures are more particularly expreſſed in Fig. 6. where A B is a Scale of Feet for the Height of the Eye, and C D a Scale of Minutes adapted thereto.

Beſides this, there is another Correction of the apparent Altitude of Objects meaſured by the Quadrant; for by Reaſon of the Refraction of Light through the Atmoſphere, or Body of Air, the Places of all Bodies are raiſed higher above the Horizon than what they really are; ſo that ſuppoſe S the true Place of the Sun or Star, its apparent Place will be at I by this Refraction of the Air. And it is the Angle or Arch G E which is meaſured by the Inſtrument, and not the Arch F E, which is the real

Altitude

Altitude above the Sea; and therefore from the measured Arch G E we subtract on one Hand the Dip of the Marine Horizon O E; and on the other, the Arch F G, the Quantity of Refraction; and then we get the Remainder F O for the true Altitude required, of the Object at S.

If we have Regard to the Zenith Distance Z F of the Object S, it is plain, that is made less by Refraction, being only Z G; therefore the Arch F G is to be added to the apparent or measured Zenith Distance, to have the true Zenith Distance Z F, of the Sun or Star at S.

The more obliquely the Rays fall on the Atmosphere, the greater will be the Refraction, and consequently the horizontal Refraction will be greatest of all, and at the Zenith, where the Rays are Perpendicular there will be no Refraction at all. The Altitude observed, must therefore be corrected by a Table of Refractions; such an one I have here added, as follows.

Ap. Alt.	Refract.	Ap. Alt.	Refract.	Ap. Alt.	Refract.	Ap. Alt.	Refract.
°	′ ″	°	′ ″	°	′ ″	°	′ ″
0	33. 0	13	3.43	26	1.47	48	0.47
1	23.50	14	3.27	27	1.42	50	0.44
2	17.43	15	3.13	28	1.38	52	0.41
3	13.44	16	3.01	29	1.34	54	0.38
4	11.05	17	2.50	30	1.30	56	0.35
5	9.10	18	2.40	32	1.23	58	0.32
6	7.49	19	2.31	34	1.17	60	0.30
7	6.48	20	2.23	36	1.12	65	0.24
8	5.59	21	2.16	38	1.07	70	0.19
9	5.21	22	2.09	40	1.02	75	0.14
10	4.50	23	2.03	42	0.58	80	0. 9
11	4.24	24	1.57	44	0.54		
12	4.02	25	1.52	46	0.50		

As the Meridian Altitude of the Sun is the common Means for finding the Latitude of the Place, it ought to be

be known with the utmoſt Accuracy; but this depends on knowing the true Quantity of Refraction in the Atmoſphere at a given Altitude, which will be variable according to the diffent Temperature and Denſity thereof at different Seaſons of the Year. This is the Reaſon why all Tables of Refraction are in ſome Degree uncertain, in ſmall Altitudes eſpecially; and thoſe given by different Authors, variable; ſome making the horizontal Refraction 35′ 30″, while others make it only 32′ 20″. We therefore chuſe to give that, which is a Mean among them all, and is nearly the ſame with that propoſed by Sir I. NEWTON.

An Example or two will make all plain. Suppoſe the Obſerver's Eye 25 Feet above the Sea, finds the Altitude of the Sun, by the Quadrant, to be 15° 30′. Now to the Elevation of the Eye A H 25 Feet, the Dip of the viſible Horizon is 6′, and to the Altitude of 15° 30′, there anſwers a Refraction in the Table 3′; the Sum of both is 9′, which deducted from the obſerved Altitude 15° 30′, leaves 15° 21′ for the true Altitude above the real Horizon, or Angle F H O.

But in Caſe of a Back-obſervation, the Proceſs will be different; becauſe as all Objects do in the lower Glaſs appear *inverted*, it muſt follow, that thoſe which are above the true Horizon will appear below it; and thoſe which are below will appear in this Glaſs above it. And, therefore, ſince upon moving the Index, the Object, whoſe Altitude is taking, will appear to riſe upwards, it will meet the Marine Horizon or Edge of the Sea, before it comes to the true Horizon in the Glaſs. To the Arch thus meaſured you muſt therefore *add* the Dip of the Sea, and from that Sum *ſubtract* the Refraction, and the Remainder will be the true Altitude of the Sun or Star.

For Example. Suppoſe by the back Obſervation, I find the Height of the Sun — — 15° 18′

The Eye's Height 25 Feet.		
Add the Dip — — —		6′
Sum	15	24
Subtract the Refraction — —		3′
Remains the true Altitude	15°	21′

as before.

N. B.

N. B. Since the Zenith Distance is the Complement of the Altitude, the Refraction is ever to be *added* to that, as it is always subducted from the horizontal Altitude.

As all Objects in the back Observation are made to ascend by moving the Index forwards, so they will apparently descend by moving the Index backwards; therefore if the Index be set to the Beginning of the Degrees, and then moved backwards, you will see the reflected Horizon, in the silvered Part of the Glass, descend to the visible Horizon or real Edge of the Sea, seen through the clear or unsilvered Part; and since the reflected Horizon is as much above the real Horizon, as the visible Horizon is below, 'tis evident, that, to make both coincide, the Index must be moved back from the Beginning of the Arch, just so much as is equal to *double the Dip of the Sea*; whence it appears, that by this Means, the Dip may be known at any Time, without measuring the Height of the Eye above the Water.

Of the DIVISION *of the* LIMB *on the* QUADRANT *by* DIAGONAL LINES, *and a* NONIUS.

AS each Degree on the Limb is divided into 3 equal Parts of 20 Minutes each, so each one of these is again subdivided into 10 others by the common Contrivance of diagonal Lines, consequently the Altitude may be taken to 2 Minutes by a Quadrant of 18 or 20 Inches Radius.

But the usual Construction of this diagonal Limb seems faulty, as it makes so great a Confusion in the Divisions by such numerous Intersections of Lines; for as there are 270 diagonal Lines, and 10 circular Ones, they make no less than 2700 Intersections, and in so small a Space, that one would wonder how such a dazzling and complicated Division of a Quadrant, in such constant and universal Use,

Uſe, could have been ſo long tolerated by Cuſtom; and nothing more verifies the Poet's Diſtich,

> CUSTOM, *which Mankind into Slavery brings,*
> *The* dull Excuſe *for doing ſilly Things.*

And it is the more to be wondered at, when we conſider how readily we find a Remedy in a Diviſion juſt as accurate and vaſtly more perſpicuous and eaſy, by diagonal Lines without any Interſections at all. For 10 Diviſions on the fiducial Edge of the Index, anſwer all the Purpoſe of 10 concentric Circles as to Accuracy, and with an hundred Times the Perſpicuity.

But the Diviſion of Degrees into Minutes, by that Contrivance called the NONIUS, is beyond Diſpute, the moſt excellent and elegant of all others; it is alſo very eaſy to be underſtood and uſed when rightly conſidered. For ſince each Degree is divided into 3 equal Parts, there will be 21 of thoſe Parts in 7 Degrees, each of which contains 20 Minutes. Then if, on that Piece of Ivory called the Nonius, we take a Length juſt equal to 7 Degrees, and divide it into 20 equal Parts, it is evident, that ſince 20 of theſe anſwers to 21 on the Limb, each one of thoſe will exceed each one on the Limb by $\frac{1}{20}$ Part, that is, by *one Minute*; therefore 2 on the *Nonius* will exceed 2 on the Limb of the Quadrant by 2 Minutes; 3 on the *Nonius*, by 3 Minutes; and ſo on. Conſequently, if the Line at the Beginning of the Diviſions on the *Nonius* be any where between two Diviſions on the Limb, there will be a Coincidence of one Diviſion on the Nonius, with one on the Limb ſomewhere; and by obſerving where that Coincidence is, you will ſee at the ſame Time how many Minutes the Index has paſſed the laſt Diviſion on the Limb. Thus if the Coincidence be at the 3d Diviſion on the *Nonius*, it ſhews the Index has paſſed the laſt Diviſion by 3 Minutes; if the Coincidence be at the 7th Line on the *Nonius*, then is the Index 7 Minutes beyond the laſt on the Limb, and ſo of the reſt.

But to make this Matter yet clearer, let A B (Fig. 7.) repreſent a Portion of the graduated Limb of the Quadrant, and C D the *Nonius* on the Index, placed appo-

sitely thereto. And in taking an Observation, suppose the Beginning of the Divisions on the *Nonius* be found between 23° and 23° 20′, as in the Figure; then looking for the first Coincidence of Lines on the *Limb* and *Nonius*, you find it to be at the 6th Division of the *Nonius*, therefore the Altitude is 23° 06′.

If two Lines on the Limb should fall within two Lines of the *Nonius* (as will sometimes happen) then that shews you must reckon $\frac{1}{2}$ a Minute, or 30″ more than the Left-hand Division denotes.

If the first Line or Index of the *Nonius* has moved over a Space less than half a Division on the Limb, then the Coincidence will be on the Right-hand in the *Nonius*; if it has moved over more than Half, it will be found on the Left-hand; if over just Half, the Coincidence will be at 10′. This shews the Reason why the Beginning 0 is placed in the Middle of the *Nonius*, and not at the End, because in such a Case you must look through all the Divisions of the *Nonius* at once for the Coincidence, whereas now we need look over but one half of them.

But if 20 Divisions on the *Nonius* be equal to 19 on the Limb, then a Division on the Limb will exceed one on the *Nonius* by $\frac{1}{20}$ Part, that is, by one Minute (for $20 : 19 :: 1 : \frac{19}{20} = 1 - \frac{1}{20}$.) Therefore the Figures of the *Nonius*, in this Case, must be placed contrary Ways, *viz.* to tell towards the Left-hand for the first 10 Minutes, and towards the Right-hand for the 10 last. This shews the Reason, why some Instruments have the Figures of the *Nonius* placed one Way, and some the other.

To find the Latitude of a Place by the Meridian Altitude, and Declination of the SUN.

THE Altitude of the Sun is taken by the Quadrant, and the Tables adjoined give the Declination of the Sun for every Day in the Year at Noon; then

then in the Diagram (Fig. 8.) you obſerve the Circle E N Q S is the Meridian; E Q, the Equator; N S, the Earth's Axis; H O, the Horizon; Z R, the prime Vertical.

Now the Sun (or Star) may be upon the Meridian at (*a*) between the South Part of the Horizon O, and the Equator E; or it may be at (*b*) between the Equator E, and the Zenith Z; or it may be on the North Part of the Meridian at (*c*) between Z, and the Pole N; or at (*d*) between the Pole and the Horizon at H. It alſo may be in the Equator at E, and in the Zenith Z; and ſo upon the Whole, there will be the following *ſix Caſes* with regard to the Latitude.

Caſe I. Let the Meridian Altitude be O *a*, and South Declination E *a*; then their Sum is O E, which taken from O Z, or 90°, leaves E Z, the Latitude of the Place.

Caſe II. Suppoſe the Sun in the Equator at E, then its Meridian Altitude O E taken from 90°, leaves the Latitude E Z.

Caſe III. Let it have North Declination at (*b*) between the Equator and Zenith; then ſubduct the Declination E *b* from the Meridian Altitude O *b*, and it will give O E, the Co-Latitude of the Place. Or, in this Caſe, the Declination E *b*, added to the Zenith Diſtance Z *b*, will give the Latitude E Z.

Caſe IV. If the Sun be exactly in the Zenith Z; then its Declination E Z, in the Table, is the Latitude of the Place.

Caſe V. If it be between the Zenith and the elevated Pole at (*c*); then the Co-Declination *c* N taken from the Meridian Altitude H *c*, leaves H N, the Pole's Height, or Latitude required.

Caſe VI. Let the Phænomenon be at (*d*) between the Pole and the Horizon; then its Meridian Altitude H *d* taken from its Declination Q *d*, leaves Q H the Complement of the Latitude ſought.

Therefore it is clear, that you can find the Latitude by Means of the Quadrant, and Table of the Declination of the Sun, which on that Account we have here ſubjoined.

Month's Day.	Jan.		Feb.		March		April		May		June	
	Sun's Declin. South.		Sun's Declin. South.		Sun's Declin. South.		Sun's Declin. North.		Sun's Declin. North.		Sun's Declin. North.	
	°	'	°	'	°	'	°	'	°	'	°	'
1	23	0	16	57	7	23	4	44	15	14	22	9
2	22	54	16	40	7	0	5	7	15	32	22	16
3	22	48	16	22	6	37	5	30	15	49	22	24
4	22	42	16	4	6	14	5	53	16	7	22	31
5	22	35	15	46	5	51	6	16	16	24	22	38
6	22	28	15	27	5	27	6	39	16	41	22	44
7	22	20	15	9	5	4	7	1	16	57	22	50
8	22	12	14	50	4	41	7	24	17	14	22	55
9	22	3	14	30	4	17	7	46	17	30	23	0
10	21	54	14	11	3	54	8	8	17	45	23	5
11	21	45	13	51	3	30	8	30	18	1	23	9
12	21	35	13	31	3	7	8	52	18	16	23	13
13	21	25	13	11	2	43	9	14	18	31	23	16
14	21	14	12	50	2	10	9	35	18	45	23	19
15	21	3	12	30	1	56	9	57	19	0	23	22
16	20	51	12	9	1	32	10	18	19	13	23	24
17	20	39	11	48	1	8	10	39	19	27	23	26
18	20	27	11	27	0	45	11	0	19	40	23	27
19	20	14	11	5	0	21	11	21	19	53	23	28
20	20	1	10	44	0N.	3	11	42	20	6	23	29
21	19	48	10	22	0	27	12	2	20	18	23	29
22	19	34	10	0	0	50	12	22	20	30	23	29
23	19	20	9	38	1	14	12	42	20	41	23	28
24	19	5	9	16	1	37	13	2	20	52	23	27
25	18	51	8	53	2	1	13	21	21	3	23	25
26	18	36	8	31	2	25	13	41	21	14	23	23
27	18	20	8	8	2	48	14	0	21	24	23	21
28	18	4	7	46	3	11	14	19	21	33	23	18
29	17	48			3	35	14	37	21	43	23	15
30	17	31			3	58	14	56	21	52	23	12
31	17	14			4	21			22	0		

Month's Day.	July		August		Sept.		Octob.		Nov.		Dec.	
	Sun's Declin. North.		Sun's Declin. North.		Sun's Declin. North.		Sun's Declin. South.		Sun's Declin. South.		Sun's Declin. South.	
	°	'	°	'	°	'	°	'	°	'	°	'
1	23	8	17	58	8	9	3	22	14	37	21	56
2	23	3	17	43	7	48	3	45	14	56	22	5
3	22	59	17	27	7	25	4	8	15	15	22	13
4	22	53	17	11	7	3	4	32	15	34	22	21
5	22	48	16	55	6	41	4	55	15	52	22	29
6	22	42	16	38	6	18	5	18	16	10	22	36
7	22	35	16	21	5	56	5	41	16	28	22	43
8	22	29	16	4	5	33	6	4	16	45	22	49
9	22	22	15	47	5	11	6	27	17	2	22	55
10	22	14	15	30	4	48	6	50	17	19	23	0
11	22	6	15	12	4	25	7	13	17	36	23	5
12	21	58	14	54	4	2	7	35	17	52	23	10
13	21	49	14	35	3	39	7	58	18	8	23	14
14	21	40	14	17	3	16	8	20	18	24	23	17
15	21	31	13	58	2	53	8	42	18	39	23	20
16	21	21	13	39	2	29	9	5	18	54	23	23
17	21	11	13	20	2	6	9	27	19	9	23	25
18	21	0	13	0	1	43	9	49	19	25	23	27
19	20	49	12	41	1	19	10	10	19	37	23	28
20	20	38	12	21	0	56	10	32	19	51	23	29
21	20	27	12	1	0	33	10	53	20	4	23	29
22	20	15	11	41	0	9	11	15	20	17	23	29
23	20	3	11	20	0 S	14	11	36	20	30	23	28
24	19	50	11	0	0	38	11	57	20	42	23	27
25	19	37	10	39	1	1	12	18	20	54	23	25
26	19	24	10	18	1	25	12	38	21	5	23	23
27	19	10	9	57	1	48	12	58	21	16	23	21
28	18	56	9	36	2	12	13	19	21	27	23	18
29	18	42	9	15	2	35	13	39	21	37	23	14
30	18	28	8	53	2	58	13	58	21	47	23	10
31	18	13	8	31			14	18			23	6

N. B. As it is most convenient, when Exactness is required, to observe the Sun's Altitude by the under or upper Limb touching the Edge of the Sea, I presume the young Navigator need not be told that in the former Case the Sun's Semidiameter, or 16′, is to be *added*, and in the latter to be *subtracted* from the observed Altitude, to have the true Altitude of the Sun's Center. And that on the Contrary, with Regard to the *Zenith Distance*, the 16′ is to be subtracted for the lower Limb, and added for the Upper, to have his true Distance from the Zenith.

A new Construction of HADLEY'S QUADRANT *with an* artificial HORIZON.

IN the Use of HADLEY'S QUADRANT, the Index moves about an Axis at Rest; if then a black Right-Line be drawn on the Index Glass A B, and placed exactly on the Center of Motion C, it will represent that Axis, and consequently *be at Rest* while the Index moves about it.

The Image of this Line in the Glass F will be plainly seen by the Eye at I, and it will there appear also without Motion, let the Quadrant or Index move as they will.

If the Plane of the Quadrant be perpendicular to the Horizon, the Axis of Motion will be parallel thereto; and therefore the apparent black Line in the Glass F will be fixed in a Position parallel to the Horizon, in every vertical Position of the Quadrant.

Lastly, in that Position of the Quadrant where the middle Point E of the Limb is in a Line C E perpendicular to the Horizon, the Line in the Glass F will be in the Plane which passes through the Eye, and is at the same Time parallel to the Horizon. Therefore the said black Line in the Glass F will become a fixed ARTIFICIAL HORIZON, as required, to render the Use of this excellent Instrument *universal* on SEA and LAND.

For

For the more ready and expeditious Uſe of this new conſtructed OCTANT, it is placed on a Foot or Stand, with a Plumb-line, or Spirit-level, to bring it perciſely into a perpendicular Poſition by adjuſting Screws at the Bottom; through the Stem there paſſes a ſtrong Screw into the back Part of the Quadrant which keeps it very ſteady and firm for Obſervation. On the Top of the Stand is a Ball and Socket on which the Quadrant is immediately placed in a *vertical*, *oblique*, or *horizontal* Poſition, as Uſe requires; and thus all Angles are meaſured from the Zenith to the Nadir, all round the Horizon, and in any oblique Circle, by proceeding according to the following Directions.

The Uſe of HADLEY'S QUADRANT *in taking* ALTITUDES *at* Land, *by Means of the* ARTIFICIAL HORIZON.

PLACE the Quadrant on a firm Table, and in a vertical Poſition with its Limb downwards, where faſten it by the Screw in the Stem, and adjuſt it by the Plumbline or Spirit-level, to a true perpendicular Poſition. Then move the Index forward till the Object is brought down to the artificial Horizon in the ſmall Glaſs, and then it will appear in the real Horizon, and the Angle of Altitude above the Horizon will be ſhewn to a Minute on the Limb of the Quadrant. This Method is moſt ready and accurate for all Objects both *Terreſtrial* and *Celeſtial*.

But with reſpect to the Sun, its Altitude may be taken, without looking at it through the Quadrant; for on the Sight-vane, which is ſilvered on the Inſide, is a black Line drawn through the Sight-hole parallel to the Horizon; therefore by only moving the Index, the Shadow of the artificial Horizon will be ſeen on the Vane, and when it exactly coincides with the black Line, the Altitude of the Sun is then ſhewn on the Limb of the Quadrant. In this Conſtruction of the Quadrant, the Moment of Time

when the upper and lower Limb of the Disk of the Sun or Moon touches the artificial Horizon, may be observed; and from thence the Time of the Transit of the whole Disk over a right Line may be known for any Altitude, which may be of frequent Use in Astronomy.

The Use of the QUADRANT *(thus constructed) for taking the Angle of* DEPRESSION below *the* HORIZON.

THE Quadrant being moveable on a Ball and Socket is to be inverted, or turned with its Arch upwards, where it is fastened by the Screw in the Stem; and adjusted to a perpendicular Situation by the Plummet or Level. Then if the Index be moved till any Object be gradually raised up to touch the *artificial Horizon*, it will then appear in the true Horizon; and the Angle of Depression, or Depth, will be expressed to a Minute on the Limb of the Quadrant above.

Thus all Angles of Altitude or Depression above and below the Horizon, from the Zenith to the Nadir, are measured with the greatest Exactness and Ease.

N. B. In all Cases, the artificial Horizon is to be understood of a Line passing thro' the Eye parallel to the Horizon of the Place. And therefore the Height of the Eye is to be added or subtracted in taking *Altitudes* or *Depths*, in the same Manner as in common Quadrants.

The Use of the QUADRANT *in measuring* HORIZONTAL ANGLES.

THE Quadrant for this Purpose is placed in a Position parallel to the Horizon, or nearly so (for Exactness in this Respect is not necessary.) Then that Line which before was the *artificial Horizon*, is now *a general Perpendicular*

cular to the Horizon. Then if there be any Number of Objects A, B, C, D, *&c.* whose angular Distances you would know, proceed thus: Bring the general Perpendicular in the Glass to coincide with the first Object A on the left Hand, and fix the Quadrant firmly; then move the Index till you see the Object B on the perpendicular Line in the Glass, and the Angle between A and B is then shewn by the Index on the Limb of the Quadrant to a Minute.

In the same Manner you proceed, by moving the Index to bring the other Objects C, D, *&c.* to the artificial Perpendicular, and their angular Distances from A, and from each other, are immediately known.

If it happens that D be more than 90 Degrees from A, and C be less, then loosen the Screw of the Socket, and turn the Quadrant round till the Object C be exactly on the Perpendicular in the Glass; then you measure the Angle between C and D, as before; which added to the Angle between A and C, gives the Angle between A and D. And thus you may proceed to measure Angles quite round the Horizon, in the same Manner as with a Theodolite, but much more exactly.

The Manner of using this new constructed QUADRANT *at* SEA.

TO the Mast of a Ship let a strong Bar of Iron be fixed, from the End of which, by means of a Gimbol, let a Pendulum be suspended; if the Ball be very heavy, and the Pendulum of 3 or 4 Feet length, the better. To this Pendulum the Quadrant is applied, and thereby kept always in a perpendicular Situation.

Then suppose the Ship in Habour, surrounded with high Lands, Clifts, Houses, *&c.* so that no Horizon appears in the Parts towards, or opposite to the Sun; in such a Case the common Use of *Hadley's Quadrant* ceases; but with the *artificial Horizon*, the Altitude of the Sun may

may be taken when it ſhines, thus; turn the Quadrant on the Pendulum to the Sun, and move the Index till you obſerve the Image of the artificial Horizon coincide with the horizontal Line on the Ivory Sight-vane, and then the Altitude of the Sun is ſhewn on the Limb of the Quadrant.

Again, in hazy or foggy Weather, when the Sun does not ſhine, and no Horizon appears, yet the Place where the Sun is, is often-times very eaſy to be obſerved, and if this be viewed in the *pendulous Quadrant*, and brought upon the artificial Horizon, the Sun's Altitude is ſhewn on the Limb nearly as exact as if it had ſhone. The Manner of making theſe Obſervations is very eaſy with a little Practice.

Or with a *ſingle Lens* the Image of the Sun, whether bright, or behind a Cloud, may be caſt on the Line of the Sight-vane, and thereby its Altitude is immediately diſcovered.

Theſe *new Quadrants* may be otherways conſtructed, to be uſed at Sea, for taking the Altitude of the Sun or Stars, when no Horizon appears. I have ſhewn them to many Gentlemen who are good Judges of the Nature and Uſes of this Inſtrument, who have unanimouſly approved of it; and to whom I have ſold many of them, of different Sizes, from 6 Inches to 2 Feet Radius.

N. B. Theſe Quadrants no ſooner were made Public, than they were pirated; but as their Truth depends upon ſome *nice Punctilios* in the Conſtruction, which they who are unacquainted with the *Theory of Optics* can know nothing of, thoſe Gentlemen who encourage the Piracy, are not to wonder if they find themſelves diſappointed in the Uſe of a bad conſtructed Inſtrument.

F I N I S.